

今さら聞けない平方完成

例題 次の2次式を平方完成せよ。

$$\begin{aligned}
 (1) \quad & x^2 - 2x - 2 \\
 &= (x-1)^2 - 1 - 2 \\
 &= \underline{(x-1)^2 - 3} \\
 &= 2(x - \frac{5}{4})^2 - \frac{25}{8} - 1 \\
 &= 2(x - \frac{5}{4})^2 - \frac{33}{8}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & -2x^2 - 8x + 1 \\
 &= -2(x^2 + 4x) + 1 \\
 &= -\frac{1}{2}(x^2 + 8x) + 3 \\
 &= -\frac{1}{2}(x+4)^2 + 8 + 3 \\
 &= -\frac{1}{2}(x+4)^2 + 11 \\
 &= \underline{-\frac{1}{2}(x+4)^2 + 9}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & -\frac{1}{2}x^2 + 4x + 3 \\
 &= -\frac{1}{2}(x^2 - 8x) + 3 \\
 &= -\frac{1}{2}(x-4)^2 + 8 + 3 \\
 &= -2(x + \frac{3}{2})^2 + \frac{9}{2} + 3 \\
 &= -2(x + \frac{3}{2})^2 + \frac{15}{2} \\
 &= \underline{-2(x + \frac{3}{2})^2 + 11}
 \end{aligned}$$

例題 次の2次式を平方完成せよ。

$$\begin{aligned}
 (1) \quad & x^2 - 8x + 2 \\
 &= (x-4)^2 - 16 + 2 \\
 &= \underline{(x-4)^2 - 14} \\
 &= -(x-1)^2 + 1 + 3 \\
 &= \underline{-(x-1)^2 + 4}
 \end{aligned}$$

$$(3) \quad -2x^2 - 6x + 3$$

$$\begin{aligned}
 &= -2(x^2 + 3x) + 3 \\
 &= -2(x + \frac{3}{2})^2 + \frac{9}{2} + 3
 \end{aligned}$$

$$(4) \quad \frac{1}{2}x^2 + 2x + 5$$

$$\begin{aligned}
 &= \frac{1}{2}(x^2 + 4x) + 5 \\
 &= \frac{1}{2}(x+2)^2 - 2 + 5
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & -x^2 + 2x + 3 \\
 &= -(x^2 - 2x) + 3 \\
 &= -(x-1)^2 + 1 + 3 \\
 &= \underline{-(x-1)^2 + 4}
 \end{aligned}$$

$$(5) \quad 2x^2 - 12ax + 2$$

$$\begin{aligned}
 &= 2(x^2 - 6ax) + 2 \\
 &= 2(x - 3a)^2 - 18a^2 + 2 \\
 &= \underline{3\{x - (a-2)\}^2 - 3(a-2)^2 + 2}
 \end{aligned}$$

$$= 3\{x - (a-2)\}^2 - 3a^2 + 12a - 12 + 2$$

$$(6) \quad 3x^2 - 6(a-2)x + b$$

$$\begin{aligned}
 &= 3\{x^2 - 2(a-2)x\} + b \\
 &= - (x^2 + 2ax) + 3a \\
 &= - (x+a)^2 + a^2 + 3a \\
 &= \underline{a(x+2)^2 - 4a + b}
 \end{aligned}$$

$$(6) \quad ax^2 + 4ax + b$$

$$\begin{aligned}
 &= a(x^2 + 4x) + b \\
 &= a(x+2)^2 - 4a + b
 \end{aligned}$$