

数と式・2次方程式 (1) 平方根がらみの有名問題

1 次の(1)~(3)の場合について、 $\sqrt{(a-1)^2} + \sqrt{(a-3)^2}$ の根号をはずし簡単にせよ。

(1) $a \geq 3$

$$\begin{aligned} \text{与式} &= |a-1| + |a-3| \\ &= (a-1) + (a-3) = \underline{2a-4} \end{aligned}$$

(2) $1 \leq a < 3$

$$\begin{aligned} \text{与式} &= |a-1| + |a-3| \\ &= (a-1) - (a-3) = \underline{2} \end{aligned}$$

(3) $a < 1$

$$\begin{aligned} \text{与式} &= |a-1| + |a-3| \\ &= -(a-1) - (a-3) = \underline{-2a+4} \end{aligned}$$

2 基礎編[1]拡大

$x = \frac{3}{\sqrt{5}+\sqrt{2}}$ 、 $y = \frac{3}{\sqrt{5}-\sqrt{2}}$ とする。次の値をそれぞれ求めよ。

(1) $x+y$ (2) xy (3) x^2+y^2 (4) x^3+y^3 (5) x^4+y^4 (6) x^5+y^5

解

$$x = \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{3(\sqrt{5}-\sqrt{2})}{3} = \sqrt{5}-\sqrt{2}$$

$$y = \frac{3(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \frac{3(\sqrt{5}+\sqrt{2})}{3} = \sqrt{5}+\sqrt{2}$$

$$(1) x+y = \underline{2\sqrt{5}}$$

$$(2) xy = \underline{3}$$

$$(3) x^2+y^2 = (x+y)^2 - 2xy = (2\sqrt{5})^2 - 2 \cdot 3 = 20 - 6 = \underline{14}$$

$$(4) x^3+y^3 = (x+y)^3 - 3xy(x+y)$$

$$= (2\sqrt{5})^3 - 3 \cdot 3 \cdot 2\sqrt{5} = 40\sqrt{5} - 18\sqrt{5} = \underline{22\sqrt{5}}$$

$$(5) x^4+y^4 = (x^2+y^2)^2 - 2x^2y^2 = 14^2 - 2 \cdot 9 = 196 - 18 = \underline{178}$$

$$(6) x^5+y^5 = (x^2+y^2)(x^3+y^3) - x^2y^3 - x^3y^2$$

$$= (x^2+y^2)(x^3+y^3) - x^2y^2(x+y)$$

$$= 14 \cdot 22\sqrt{5} - 9 \cdot 2\sqrt{5}$$

$$= 308\sqrt{5} - 18\sqrt{5} = \underline{290\sqrt{5}}$$