

整数 (3) 1次不定方程式② 「ユークリッドの互除法」

【例題1】 [基礎編 41]

ユークリッドの互除法を利用して、703と1501の最大公約数を求めよ。

$$\begin{array}{r} 2 \\ 703 \overline{) 1501} \\ \underline{1406} \\ 95 \end{array} \quad \begin{array}{r} 7 \\ 95 \overline{) 703} \\ \underline{665} \\ 38 \end{array} \quad \begin{array}{r} 2 \\ 38 \overline{) 95} \\ \underline{76} \\ 19 \end{array}$$

$$\textcircled{1501}^9 = \textcircled{703}^4 \times 2 + \textcircled{95}^6$$

$$\textcircled{703} = \textcircled{95} \times 7 + \textcircled{38}$$

$$\textcircled{95} = \textcircled{38} \times 2 + \textcircled{19}$$

$$\textcircled{38} = \textcircled{19} \times 2$$

よ? G.C.M は 19

【1】 次の2つの整数の最大公約数を、互除法を用いて求めよ。

(1) 667, 299

$$\begin{array}{r} 2 \\ 299 \overline{) 667} \\ \underline{598} \\ 69 \end{array} \quad \begin{array}{r} 4 \\ 69 \overline{) 299} \\ \underline{276} \\ 23 \end{array}$$

$$\textcircled{667} = \textcircled{299} \times 2 + \textcircled{69}$$

$$\textcircled{299} = \textcircled{69} \times 4 + \textcircled{23}$$

$$\textcircled{69} = \textcircled{23} \times 3$$

G.C.M は 23

(2) 779, 391

$$\begin{array}{r} 1 \\ 391 \overline{) 779} \\ \underline{391} \\ 388 \end{array}$$

$$\textcircled{779} = \textcircled{391} \times 1 + \textcircled{388}$$

$$\textcircled{391} = \textcircled{388} \times 1 + \textcircled{3}$$

$$\textcircled{388} = \textcircled{3} \times 129 + \textcircled{1}$$

G.C.M は 1

例題2 等式  $4x + 29y = 2$  を満たす整数  $x, y$  の組を1つ求めよ。

$$\begin{array}{r} 29 \overline{) 42} \\ \underline{29} \phantom{0} \\ 13 \phantom{0} \end{array}$$

$$\begin{aligned} 42 &= 29 \cdot 1 + 13 & 13 &= 42 - 29 \cdot 1 \\ 29 &= 13 \cdot 2 + 3 & \Rightarrow 3 &= 29 - 13 \cdot 2 \\ 13 &= 3 \cdot 4 + 1 & 1 &= 13 - 3 \cdot 4 \end{aligned}$$

$$\begin{aligned} 1 &= 13 - 3 \cdot 4 \\ &= 13 - (29 - 13 \cdot 2) \cdot 4 \\ &= 13 \cdot 9 + 29 \cdot (-4) \\ &= (42 - 29 \cdot 1) \cdot 9 + 29 \cdot (-4) \\ &= 42 \cdot 9 + 29 \cdot (-13) \end{aligned}$$

よって  $42 \cdot 18 + 29 \cdot (-26) = 2$

よって特殊解  $(x, y) = (18, -26)$

2 次の等式を満たす整数  $x, y$  の組を1つ求めよ。

(1)  $31x + 22y = 1$

$$\begin{aligned} 31 &= 22 \cdot 1 + 9 & 9 &= 31 - 22 \cdot 1 \\ 22 &= 9 \cdot 2 + 4 & \Rightarrow 4 &= 22 - 9 \cdot 2 \\ 9 &= 4 \cdot 2 + 1 & 1 &= 9 - 4 \cdot 2 \end{aligned}$$

$$\begin{aligned} 1 &= 9 - 4 \cdot 2 \\ &= 9 - (22 - 9 \cdot 2) \cdot 2 \\ &= 9 \cdot 5 + 22 \cdot (-2) \\ &= (31 - 22 \cdot 1) \cdot 5 + 22 \cdot (-2) \\ &= 31 \cdot 5 + 22 \cdot (-7) \end{aligned}$$

よって  $(x, y) = (5, -7)$

(2)  $31x + 22y = 3$

よって  $31 \cdot 15 + 22 \cdot (-21) = 3$

$(x, y) = (15, -21)$

※対策編 [24] に取り組もう!