

三角関数 (4) 三角関数の合成②

例題1 $0 \leq x < 2\pi$ のとき、不等式 $\sin x + \sqrt{3} \cos x < 1$ を解け。

$$2 \sin(x + \frac{\pi}{3}) < 1$$

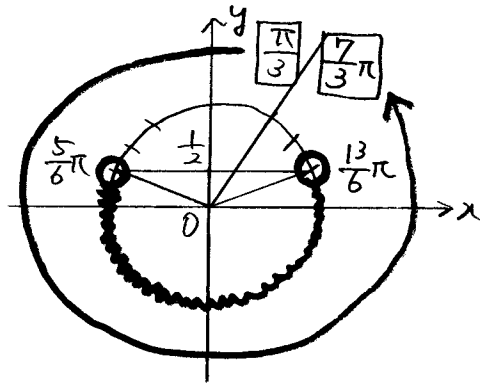
$$\sin(x + \frac{\pi}{3}) < \frac{1}{2}$$

角の変域

$$\frac{\pi}{3} \leq x + \frac{\pi}{3} < 2\pi + \frac{\pi}{3}$$

$$\text{よ} \therefore \frac{5}{6}\pi < x + \frac{\pi}{3} < \frac{13}{6}\pi$$

$$\frac{\pi}{2} < x < \frac{11}{6}\pi$$



1 $0 \leq x < 2\pi$ のとき、次の不等式を解け。

(1) $\sin x - \cos x \geq 1$

$$\sqrt{2} \sin(x - \frac{\pi}{4}) \geq 1$$

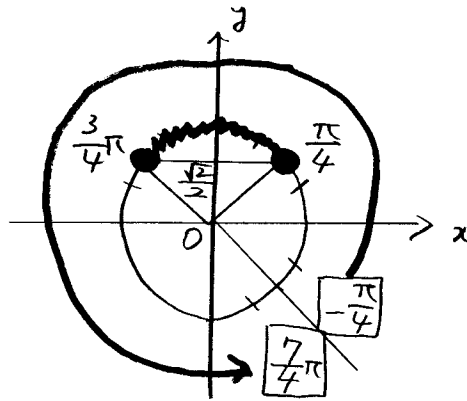
$$\sin(x - \frac{\pi}{4}) \geq \frac{\sqrt{2}}{2}$$

角の変域

$$-\frac{\pi}{4} \leq x - \frac{\pi}{4} < 2\pi - \frac{\pi}{4}$$

$$\text{よ} \therefore \frac{\pi}{4} \leq x - \frac{\pi}{4} \leq \frac{3}{4}\pi$$

$$\frac{\pi}{2} \leq x \leq \pi$$



(2) $\sqrt{3} \sin x + \cos x \leq \sqrt{3}$

$$2 \sin(x + \frac{\pi}{6}) \leq \sqrt{3}$$

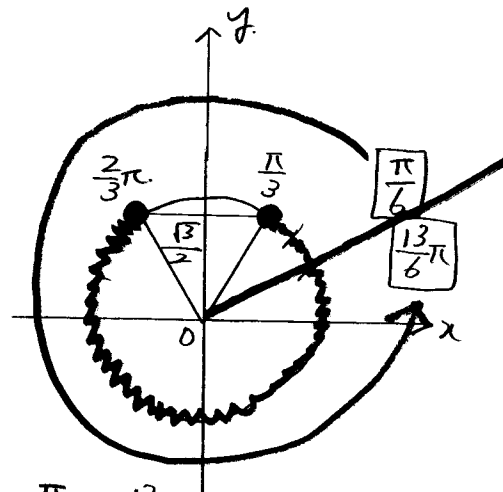
$$\sin(x + \frac{\pi}{6}) \leq \frac{\sqrt{3}}{2}$$

角の変域

$$\frac{\pi}{6} \leq x + \frac{\pi}{6} < 2\pi + \frac{\pi}{6}$$

$$\text{よ} \therefore \frac{\pi}{6} \leq x + \frac{\pi}{6} \leq \frac{\pi}{3}, \frac{2}{3}\pi \leq x + \frac{\pi}{6} < \frac{13}{6}\pi$$

$$0 \leq x \leq \frac{\pi}{6}, \frac{\pi}{2} \leq x < 2\pi$$



例題2 関数 $y = \sin x + \cos x$ ($0 \leq x < 2\pi$) の最大値と最小値、およびそのときの x の値を求めよ。

$$y = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$$

角の変域

$$\frac{\pi}{4} \leq x + \frac{\pi}{4} < 2\pi + \frac{\pi}{4}$$

$$-1 \leq \sin\left(x + \frac{\pi}{4}\right) \leq 1 \text{ だから}$$

$$\text{Max } \sqrt{2} \left(x + \frac{\pi}{4} = \frac{\pi}{2} \text{ かつ } x = \frac{\pi}{4}\right)$$

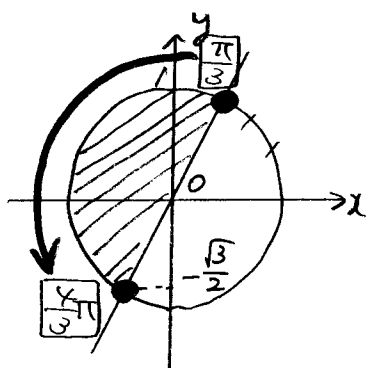
$$\text{Min } -\sqrt{2} \left(x + \frac{\pi}{4} = \frac{3}{2}\pi \text{ かつ } x = \frac{5}{4}\pi\right)$$

2 $0 \leq x \leq \pi$ のとき、関数 $y = \sin x + \sqrt{3} \cos x$ の最大値と最小値、およびそのときの x の値を求めよ。

$$y = 2 \sin\left(x + \frac{\pi}{3}\right)$$

角の変域

$$\frac{\pi}{3} \leq x + \frac{\pi}{3} \leq \frac{4}{3}\pi$$



$$-\frac{\sqrt{3}}{2} \leq \sin\left(x + \frac{\pi}{3}\right) \leq 1 \text{ だから}$$

$$\text{Max } 2 \left(x + \frac{\pi}{3} = \frac{\pi}{2} \text{ かつ } x = \frac{\pi}{6}\right)$$

$$\text{Min } -\sqrt{3} \left(x + \frac{\pi}{3} = \frac{4}{3}\pi \text{ かつ } x = \pi\right)$$

3 θ の関数 $y = \sin 2\theta + \sin \theta + \cos \theta$ について

(1) $t = \sin \theta + \cos \theta$ とおいて、 y を t の関数で表せ。

$$t^2 = \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$$

$$t^2 = 1 + \sin 2\theta \quad \therefore \sin 2\theta = t^2 - 1$$

$$\therefore y = (t^2 - 1) + t \quad \therefore \boxed{y = t^2 + t - 1}$$

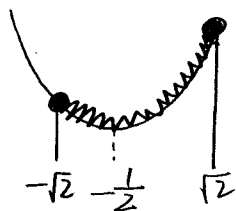
(2) t のとりうる値の範囲を求めよ。

$$t = \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right)$$

$$\therefore \boxed{-\sqrt{2} \leq t \leq \sqrt{2}}$$

(3) y の最大値、最小値を求めよ。

$$y = \left(t + \frac{1}{2}\right)^2 - \frac{5}{4}$$



$$\text{Max } 1 + \sqrt{2}$$

$$\text{Min } -\frac{5}{4}$$