Σの公式

① $\displaystyle\sum_{k=1}^{n} k=\frac{n(n+1)}{2}$ ② $\displaystyle\sum_{k=1}^{n} k^2=\frac{n(n+1)(2n+1)}{6}$ ③ $\displaystyle\sum_{k=1}^{n} k^3=\left\{\frac{n(n+1)}{2}\right\}^2$

④ $\displaystyle\sum_{k=1}^{n} c=nc$

例題１ 次の和を求めよ。

(1) $\displaystyle\sum_{k=4}^{15} k^2=\sum_{k=1}^{15} k^2-\sum_{k=1}^{3} k^2$

$$=\frac{\cancel{15}^{5}\cdot\cancel{16}^{8}\cdot 31}{\cancel{6}\,\cancel{2}\,1}-(1+4+9)$$

$$=1240-14$$

$$=\underline{1226}$$

(2) $\displaystyle\sum_{k=1}^{n} 2^{k-1}=\frac{2^n-1}{2-1}$

$$=\underline{2^n-1}$$

(3) $\displaystyle\sum_{k=1}^{n}(3k^2-7k+4)=3\cdot\frac{n(n+1)(2n+1)}{6}-7\cdot\frac{n(n+1)}{2}+4n$

$$=\frac{n(n+1)(2n+1)-7n(n+1)+8n}{2}$$

$$=\frac{n\{(n+1)(2n+1)-7(n+1)+8\}}{2}$$

$$=\frac{n(2n^2-4n+2)}{2}$$

$$=n(n^2-2n+1)=\underline{n(n-1)^2}$$

(4) $1^2\cdot2+2^2\cdot3+3^2\cdot4+\cdots\cdots+n^2(n+1)=\displaystyle\sum_{k=1}^{n}k^2(k+1)=\sum_{k=1}^{n}(k^3+k^2)$

$$=\frac{n^2(n+1)^2}{4}+\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{3n^2(n+1)^2+2n(n+1)(2n+1)}{12}$$

$$=\frac{n(n+1)\{3n(n+1)+2(2n+1)\}}{12}$$

$$=\frac{n(n+1)(3n^2+7n+2)}{12}=\boxed{\frac{n(n+1)(n+2)(3n+1)}{12}}$$

1 次の和を求めよ。

(1) $\sum_{k=1}^{n}(3^k+2) = \dfrac{3(3^n-1)}{3-1} + 2n = \boxed{\dfrac{3^{n+1}-3+4n}{2}}$

(2) $\sum_{k=1}^{n}(5k^2-4k+2)$

$= 5\cdot\dfrac{n(n+1)(2n+1)}{6} - 4\cdot\dfrac{n(n+1)}{2} + 2n$

$= \dfrac{5n(n+1)(2n+1) - 12n(n+1) + 12n}{6}$

$= \dfrac{n\{5(2n^2+3n+1) - 12(n+1) + 12\}}{6}$

$= \boxed{\dfrac{n(10n^2+3n+5)}{6}}$

(3) $(1+1^3)+(2+2^3)+(3+3^3)+\cdots\cdots+(n+n^3)$

$= \sum_{k=1}^{n}(k+k^3)$

$= \dfrac{n(n+1)}{2} + \dfrac{n^2(n+1)^2}{4}$

$= \dfrac{2n(n+1) + n^2(n+1)^2}{4}$

$= \dfrac{n(n+1)\{2+n(n+1)\}}{4} = \boxed{\dfrac{n(n+1)(n^2+n+2)}{4}}$

2 次の数列の第 k 項を求めよ。また，初項から第 n 項までの和を求めよ。

1，1+2，1+2+3，……，$\boxed{1+2+3+\cdots\cdots+n}$，……

(第n項) $= \sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$

よって 第k項は $\boxed{\dfrac{k(k+1)}{2}}$

$S_n = \sum_{k=1}^{n}\dfrac{k(k+1)}{2}$

$= \sum_{k=1}^{n}\left(\dfrac{1}{2}k^2 + \dfrac{1}{2}k\right)$

$= \dfrac{1}{2}\cdot\dfrac{n(n+1)(2n+1)}{6} + \dfrac{1}{2}\cdot\dfrac{n(n+1)}{2}$

$= \dfrac{n(n+1)(2n+1) + 3n(n+1)}{12}$

$= \dfrac{n(n+1)(2n+1+3)}{12}$

$= \dfrac{n(n+1)(2n+4)}{12}$

$= \boxed{\dfrac{n(n+1)(n+2)}{6}}$